Markov Data Game

Teacher Facilitation Guidelines

If you have not done so already, please watch the short student and teacher videos, and play *Markov* before or as you read these notes. Also, you should look at the student worksheet for *Markov*.

Learning Goals

- Students develop their understanding of how to interpret and apply data from two-way frequency tables, and develop their probabilistic reasoning skills. (The relevant CCSSM standards are listed at the end of this document.)

Prior to Students Playing

- Background information for you that students don't need to know:
    - This game is based on a "Markov process." Markov processes (also called Markov chains) appear in discrete math, systems analysis, game theory, and Economics. A Markov chain is a network that connects the states of a system; the system can transition from one state to another based on probabilities. In this game, the "state" is defined by Markov's previous two moves. Using this model, therefore, the next action is not based on the entire history of the process, but only on the current state: a small number of previous actions.
- For suggestions on how to prepare to play Data Games with students, go to the Teacher FAQ section of the Data Games website (http://play.ccssgames.com/faq-page).

During Gameplay

- **Starting on Level One: Tethys**
    - When students first begin playing, they will mostly just be guessing, so they will lose several games. This is a helpful phase, however, because they're collecting data; but some students won't like to hear Madeline yelp when they lose! Encourage them to learn from their losses in order to develop a winning approach.
    - The default graph for this game is a dot plot showing categorical data, analogous to a two-way frequency table. It is a different style of graph than many students have seen before, so student understanding may require extra teacher support.
    - If students get stuck in *Markov,* it is usually because they have not yet figured out how to interpret the graph. You should reinforce the importance of understanding the graph in beating Markov! It is the most valuable tool they have for examining the data to see Markov's patterns.
    - Another place that students might get stuck is in not recognizing that Markov's moves do not depend at all on students' moves, but instead only on his own two previous moves. This is not what students generally expect, and many of them will persist in looking for patterns between Markov's and their moves. You can point out to them, as needed, that neither the game

screen nor the graph shows the history of students' moves. The student worksheet also prompts students to notice that their moves are not being shown in the graph. (Student moves are shown in the table.)

- Students do not need to click *Clear Data* between games in the first Tethys level, as Markov will continue the same patterns for all games in this level. In fact it helps students to continue accumulating data so they can better predict Markov's moves. Students should, however, click *Clear Data* when they switch to a different level.
- Students might be puzzled at first by Markov not being perfectly consistent in his patterns. But analyzing the graph will lead students to naturally begin using language such as "most likely" to predict what Markov will do next, and thus begin using probabilistic reasoning. Reinforce this reasoning by providing prompts such as "So you can see that Markov isn't always perfectly predictable, because he's evil, right? But can you guess what he is most likely to do next?"
- Once students have figured out Markov's patterns in order to predict his next move, they might then make the mistake of choosing what Markov's next move will be, rather than what their next move will be. At no point in the game do students actually enter Markov's moves—they always use what they predict Markov's move will be to enter their move.
- Discourage students from bouncing around and playing different levels of the game, instead of persisting in the first level until they can defeat Markov consistently. Similarly, they shouldn't click *Set Strategy* until they have figured out how to beat Markov.
- The first two moves of any game at any level in *Markov* will always be guesses, since the student can't see Markov's previous two moves.

- **Set Strategy**
  - Students should watch the second video after they can beat Markov consistently. The video shows them how to navigate the Set Strategy screen.
  - Even with a terrific strategy, students could lose a game here or there because of the probabilistic nature of the game!
  - When students go to the Set Strategy screen, they don't have to specify a complete strategy for all nine possibilities if they don't yet have enough data; if they haven't set automatic moves for some situations, the game will stop and allow them to enter their moves manually. As they collect more data, they can go back and fill in more boxes or revise their strategy.
  - Once they have devised and successfully used a strategy several times, students can further speed up the game by weighting their moves. They will use informal conditional probability analysis, based on how confident they are of Markov's next moves. So at this point, the game is not just "Can you beat Markov?", but "How fast can you beat Markov?!"
  - The number of weights in each of the strategy boxes is the number of steps Madeline moves up or down after that move. Students may use the 18 weights shown to distribute among the 9 boxes, with at least one weight remaining in each box. So if students are really sure about "RR," for example, they can weight this move more heavily; whereas if they have no data about "RS," they can leave just one weight there.

- **Possible Extensions**
  - Each possible combination of Markov's previous two moves doesn't show up on the horizontal axis until that combination occurs during a game. (This also applies to Markov's current move on the vertical axis.) Students might notice this, so you might pose a little extension question: "How many total possible combinations of Markov's previous two moves are there?" (**Nine**) "How do you know?"
  - After students have set a strategy and weighted their moves to their satisfaction in the Tethys level, they can go on to try other levels of the game. They will need to click *Levels*, which will appear after they end a game. The student worksheet only guides students through the first Tethys level, but they should have a good idea how to proceed on their own.

Answers to Student Worksheet Questions

- "(Q1) Take a good look at the data in the graph to see how it can help you understand Markov's patterns. Are your moves being shown in the graph?" – **No**
- "(Q2) Suppose you were asked to help out a friend who has played the game but hasn't yet figured out how to defeat Markov. Explain below to your friend how to use the data and win (without going to the Set Strategy screen)." – **Explanations will vary, but should include looking at Markov's two previous moves, analyzing the graph to see what he is most likely to do next, and choosing the move that will defeat his most likely move.**
- "(Q3) Playing against Markov is frustrating sometimes, as he is not always consistent in his pattern of moves, right? Explain to your friend how you decided to make some of your moves even when you're not absolutely sure what Markov will do, based on looking at the data from previous games. Give an example from your data." – **Use the graph to predict what move Markov is most likely to make, based on the most frequent move he has made in the current situation with his previous two moves. If two or more moves are tied in frequency, then you can reasonably choose either of his moves.**

Challenges Introduced on Each Level

- Markov uses a different strategy on each new level, but his strategy remains consistent throughout each game played on a single level.
- There is increasing variability in Markov's moves as the student progresses through the levels, which makes it more challenging to win.
- There are no other gameplay differences between the levels.

Relevant Common Core State Standards for Mathematics

- Investigate patterns of association in bivariate data (8.SP.4)
- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.5)

- Use probability to evaluate outcomes of decisions (S.MD.5)
- Make sense of problems and persevere in solving them (Standard for Mathematical Practice - 1)
- Model with mathematics (Standard for Mathematical Practice - 4)
- Use appropriate tools strategically (Standard for Mathematical Practice - 5)
- Attend to precision (Standard for Mathematical Practice - 6)